# Transactions Paper

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## THE NATURAL FREQUENCY OF PARALLEL CAPACITOR BANKS


W. H. Cuttino
Member AIEE

Miles Maxwell
Associate Member AIEE

Both of:
Westinghouse Electric Corp.
East Pittsburgh, Pa.



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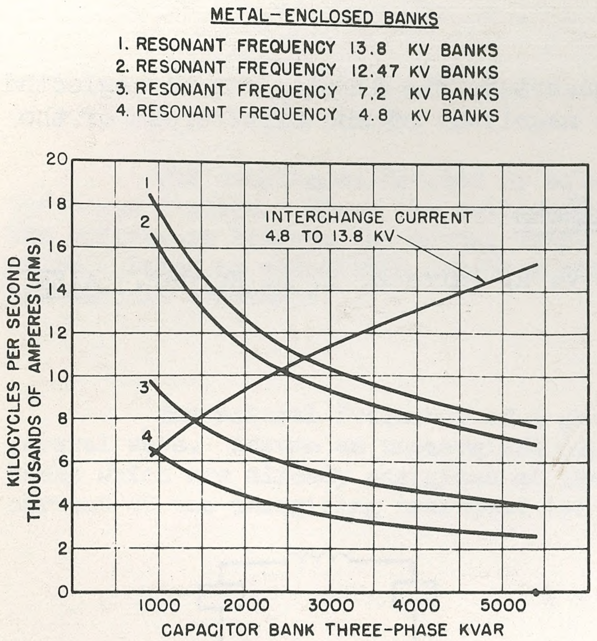
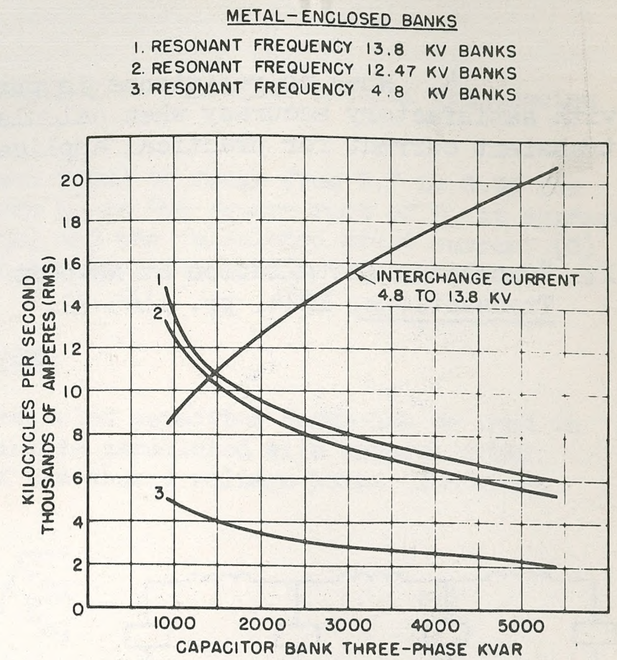



Figure 4 - Transient interchange current and frequency when energizing one capacitor bank with no initial charge in parallel with an energized bank of the same size.

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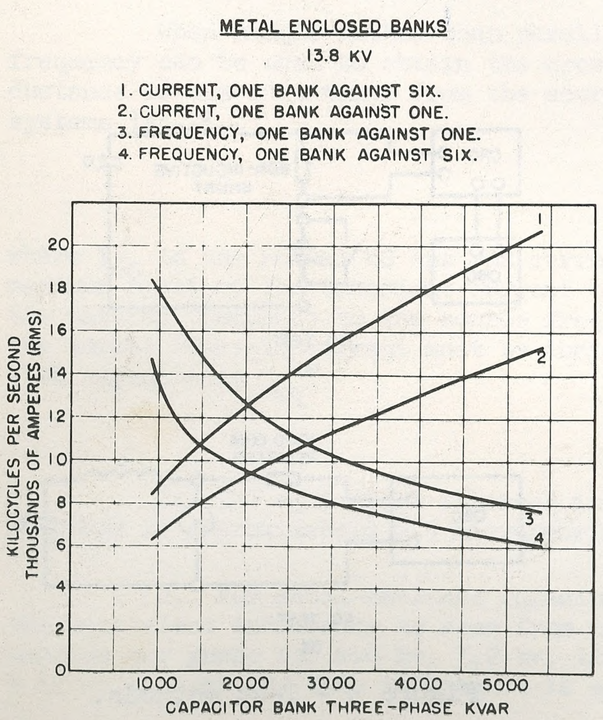



Figure 6 - Comparison of one bank against one and one bank against six.

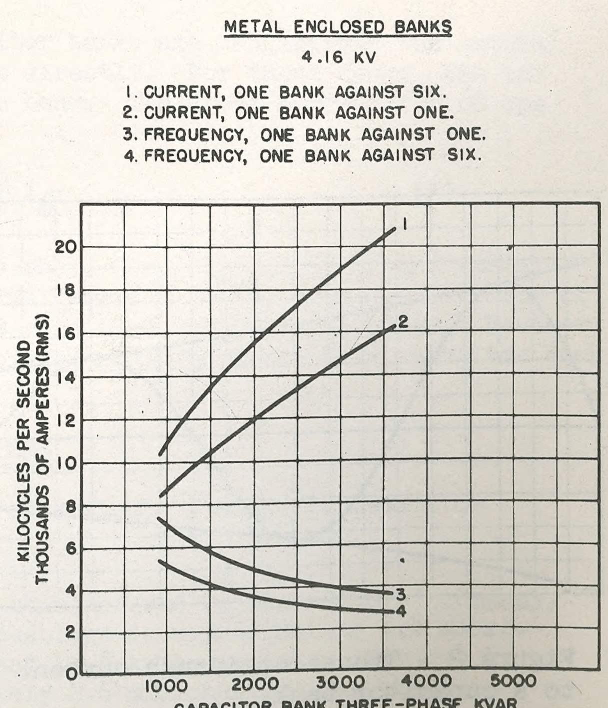



Figure 7 - Transient current and frequency of 4.16 kv metal-enclosed capacitor banks.

# THE NATURAL FREQUENCY OF PARALLEL CAPACITOR BANKS

W. H. Cuttino*
Member AIEE

Miles Maxwell**
Associate Member AIEE

The magnitude and natural frequency of the transient currents which flow through the switching device when parallel banks of capacitors are switched is of great importance in determining the severity of the duty on the switching device. It is also important to know the magnitude and natural frequency of the transient current that will flow to the combination of individual fuse and faulted capacitor when evaluating the effects from the stored energy from a large number of capacitors closely connected in parallel.

It can be assumed that a capacitor is most likely to fault at the peak of the voltage wave which means that the parallel capacitors are charged to peak voltage. This stored energy will attempt to drain off through the faulted capacitor unless the fuse interrupts the circuit before all the energy has been dissipated. If the number of capacitor units in parallel are increased to a large enough number, the stored energy will be great enough to rupture the case of the faulted capacitor. For banks of this size, it is desirable to provide an individual capacitor fuse which will interrupt the circuit before all the energy is dissipated or arrange the capacitor bank so that the number of capacitors in parallel in a given group will be limited to a value which can be satisfactorily handled by a less expensive lower interrupting rating individual fuse. It is therefore desirable to establish the order of transient current magnitude and natural frequency for the various capacitor assemblies to make sure that the laboratory set-up for testing fuses and capacitor case rupture are in the ranges expected in actual installations.

In recent years the closely-coupled switched capacitor banks have been increasing greatly in number and size. Since the natural frequency and bank sizes are considered important factors involved in the performance and maintenance of the switching device, it is desirable that their effect on a device intended for parallel capacitor switching be established.

The larger capacitor banks are usually made up of either metal-enclosed equipments or open-stack type equipments. It is therefore desirable to establish the probable range of natural frequencies for various bank sizes and voltage rating equipments for the two types of equipments, particularly the larger bank sizes which are closely coupled as they represent the more severe duty. When testing a device for capacitor switching, it is therefore important that not only the kvar size and voltage rating of the capacitor banks be maintained, but the inductance should be kept within limits to cover the natural frequency range that may be encountered, particularly the higher natural frequency conditions obtained with minimum inductance between banks.

In the interest of economy and flexibility, the capacitor banks in high power laboratories are arranged in groups for series-parallel connection to permit many types of tests at many voltages. This requires spacing the component parts at distances great enough to permit tests at the upper voltage limit. When arranging the laboratory bank set-up for parallel capacitor switching tests, particularly for voltages in the range of 4 to 15 kv, it is necessary to take precautions in arranging the connections for low inductance if the most severe conditions expected in actual installations are to be met. It is therefore necessary to know the natural frequency of the bank sizes contemplated

* Distribution Apparatus Engineering, Westinghouse Electric Corporation, East Pittsburgh, Pennsylvania.
** Electric Utility Engineering, Westinghouse Electric Corporation, East Pittsburgh, Pa.



for the most severe actual installations to be certain the laboratory tests adequately test the switching device.

Methods have been described[1] previously for determining the natural frequencies of capacitor banks made up of 15 kvar capacitor units which were not very closely coupled. The early field tests were made on open-type assemblies with considerable spacing between banks. More recent installations have involved large metal-enclosed equipments with 25 kvar capacitor units where the external inductance between parallel banks was less than 4 microhenries. In these cases the internal inductance of the capacitor equipment plays an important factor in establishing the magnitude and natural frequency of the transient currents during parallel switching operations.

Recently the company represented by the authors has undertaken a comprehensive program to study the effects of switching extremely closely-coupled capacitor banks on the switching devices. In conjunction with this, a series of tests were made to determine the natural frequencies of these closely-coupled parallel banks and to develop means for calculating natural frequencies. This paper presents the results of these natural frequency tests.

## THEORY

Capacitor banks are made up of individual capacitors of relatively small size. The actual exact representation of such a bank would require a complicated ladder network.

Measurements have shown that it is generally possible to represent such a capacitor bank as a lumped capacitance and inductance. By the same token, tests have shown that it is possible to represent several parallel capacitor banks as a single capacitance and inductance. Figure 1-A shows the approximate representation of four parallel capacitor banks. Note that no power source connection is shown. In order to get maximum values of inrush currents, the voltages used in tests are timed so that the capacitors are energized at the crest of the voltage wave. When the natural frequency of the capacitor bank is above about 600 cps, the energizing voltage may be assumed to be a d-c voltage equal to the crest voltage of the source voltage. Due to decay of the energizing transient, the energizing inrush has died out long before the 60 cps current reaches its crest value. If the natural frequency of the capacitor bank being considered approaches the frequency of the source voltage, most of the equations used in this paper do not apply. However, for the practical range of present-day applications, the natural frequencies are well above the source frequency.

In Figure 1-B, the reduction of the four banks of Figure 1-A is shown. The calculation of the equivalent capacitance and inductance is of necessity an approximation. The most accurate method for this type of problem is to assume that the capacitors to be lumped are paralleled at the point where the internal inductances are neglected as shown by the dash line in Figure 1-A. The equivalent capacitance is then the sum of the capacitances and the equivalent inductance is obtained by a simple reduction of the inductive network. This method of reduction gave results which agreed within plus or minus 5% of test values.

The natural frequency of a group of parallel capacitors is:

$$f_n = \frac{1}{2\pi\sqrt{L_{eq}\, C_{eq}}} \qquad (1)$$

Where $L_{eq}$ or $C_{eq}$ are the equivalent constants for the particular case to be studied, the peak-to-peak damping of the transient inrush current (see Figure 2) is:

$$D = \frac{B}{A} = 1 - 0.5 \pi R \sqrt{\frac{C}{L}} \qquad (2)$$

Equation 2 is an approximate equation, which is nearly exact if $1/LC$ is more than approximately five times $R^2/4L^2$. Unless resistance of appreciable magnitude has been introduced into the circuit purposely, equation 2 holds for practical applications. In the d-c tests, equation 2 was checked by calculating the exact circuit characteristics from oscilloscope pictures. These checks show reasonably good accuracies for equation 2.

When a capacitor bank is charged with a d-c voltage and then discharged, the frequency of the discharge is:

$$f_{ac} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \qquad (3)$$

If $R^2/4L^2$ is small compared to $1/LC$, the frequency of the discharge current is the same as the natural frequency of the bank as given by equation 1.

## TEST METHODS

In order to obtain better data for use in testing and applying switching devices to parallel capacitor banks, a series of field and laboratory tests were made. There were two types of measurements. In the d-c test method, a capacitor bank was charged with d-c voltage and discharged through a non-inductive shunt. The voltage across the shunt was fed into a cathode ray oscilloscope. In addition, a timing wave was fed into the oscilloscope and both traces appeared on the screen. A photograph of the screen was taken the same instant the capacitor bank was discharged. The discharge frequency was determined by comparing the period of the oscillatory discharge with the period of the timing wave.

The frequency of the oscillatory discharge is given by equation 3. Figure 3-A shows the test set-up.

In the a-c test, a variable frequency oscillator and a vacuum-tube voltmeter were used. Since these were steady-state tests, resistance could not affect the frequency so a 1000-ohm resistor was used as shown in Figure 3-B in order to protect the oscillator.

## TESTS

Tests were made on both open-stack type capacitor banks and metal-enclosed capacitor banks. In actual installations, the parallel banks of metal-enclosed capacitors usually have the highest natural frequencies and, therefore, offer the most severe duty on switching devices. This follows from the fact that the metal-enclosed banks are more compact and the switching devices are enclosed in the same housing. In stack-type installations the lead length involved in incorporating the switching devices in the circuit are usually of fairly appreciable length compared to the comparable lengths in metal-enclosed units.



## ACCURACY

The frequency measurements were used to calculate the equivalent inductance of the banks considering the equivalent capacitance to be the total lumped capacitance. This was done since the capacitance varies somewhat from bank to bank even among banks with the same nominal kilovar and voltage rating.

For multiple stack-type capacitor banks and metal-enclosed banks, the inductance values checked within about plus or minus 5% between different banks and the two different test methods. The frequencies ranged from approximately 2 to 15 kc for the banks tested.

For a single stack-type assembly, the natural frequency is of the order of 30 to 50 kc. Calculations made with the assumption that all the capacitor units were lumped at a point midway down the assembly indicated an equivalent inductance of about 2.0 microhenries (uh) when measured from the end of the unit. Tests made with the a-c method indicated equivalent inductances of 0.5 uh. The d-c method gave 0.9 uh on the same assembly. This is apparently a large difference. However, despite the fact the minimum length connections were used for the tests, there was some inductance introduced by the shorting switch in the d-c tests. This extra inductance was calculated to be 0.3 uh which would correct the d-c results to 0.6 uh. These results would then be within plus or minus 10% of the average of the two.

## TEST RESULTS

As mentioned above, the tests show that the indicated equivalent inductance of a single stack-type capacitor assembly is approximately 0.5 uh. This shows that it is incorrect to assume the capacitors to be lumped at some point other than very near the end of the unit. In most cases, the accuracy of the calculations of the inductance of the connections is such as to make the 0.5 uh internal inductance negligible. Therefore, the inclusion of the inductance of the leads used to make up the various phase groups when more than one stacking unit per phase is used is sufficient. The assumptions made in reference 1 which omit all inductance except the connections between the capacitor bank and the bus are sufficiently accurate for most open-stack type capacitor installations.

The tests on metal-enclosed banks gave the equivalent inductance as seen from the bushings on top of the housing. Again, this inductance could be accounted for by the calculated inductance for the leads used inside the housing to get from the beginning of the horizontal buses in the capacitor compartment, through the breaker to the roof bushings. For the 4.8 kv, 7.2 kv, 12.47 kv, and 13.8 kv equipments, the inductance measured approximately 4.4 uh. The low inductance is taken in order to be conservative. The range was 4.4 to 4.8 uh. Calculations of internal inductance neglecting the inductance in the horizontal buses to which the capacitor units are connected accounts for very close to 4.4 uh.

The test also showed that the lumping of inductance and capacitance to an equivalent is accurate. There are many other frequencies associated with these transient currents. There are oscillations between the capacitor units in one group, between different groups in the same bank and between banks. However, in both the d-c and a-c tests there was a definite frequency associated with the particular point of measurement. In the d-c tests no evidence of distortion of the predominant frequency discharge current due to extraneous frequencies was noted. Comparison of calculations and the results of

both types of tests lend confidence to the methods used to calculate natural frequencies of parallel capacitor banks.

The damping, as defined by equation 2, was found to range from 0.7 to 0.95 for the cases studied. Since the ratio A/C (see Figure 2) is the square root of D, it appears the difference between the actual crest current (A) and the calculated crest current (C) as calculated by equation 4 can be neglected with accuracy comparable to the accuracy of the circuit constants.

## APPLICATIONS

The natural frequency of a group of paralleled capacitor banks can be used in several ways. Assume an unenergized capacitor bank is paralleled with several other banks which are already energized at the crest of the source voltage wave. The crest current of the energizing transient is:

$$I = \sqrt{2}\, E_{L-N} \sqrt{\frac{C_{eq}}{L_{eq}}} \qquad (4)$$

This may be referred to as the normal maximum natural frequency crest current. If the bank added is already charged to crest voltage when paralleled, and this charge is of opposite polarity to the source voltage the crest transient current is twice that given in equation 4. This may occur if a bank is switched off and then back on before its charge has decayed. If a single re-strike occurs at the worst point when a bank is being switched off, again twice the normal current can occur. A second re-strike can give currents of four times the normal maximum.

Figures 4, 5, 6, and 7 show curves of natural frequency and normal maximum crest current assuming no re-strikes or residual charge for a number of combinations of paralleled metal-enclosed capacitors. The frequency and crest current of the same size stack-type banks would normally be lower.

When single rather than parallel capacitor banks are considered, the natural frequency can be used to obtain the crest current directly. For these cases, the inductance is the inductance from the source to the bank. The crest current for 60 cps systems is:

$$I = \sqrt{2}\, \frac{f_n}{60}\, I_{60} \qquad (5)$$

where $I_{60}$ is the normal 60 cps rms current. This fact; i.e., the ratio of the normal maximum crest of the transient current to the normal current is the same as the ratio of the natural frequency to the source frequency, is also true for parallel banks. However, the normal 60-cycle current must be that current which would be drawn by a capacitor bank with capacitance $C_{eq}$.

## CONCLUSIONS

1. The equivalent internal inductance of a single stack-type capacitor assembly is approximately 0.5 microhenries.

2. For metal-enclosed capacitor banks manufactured by the authors' company, the equivalent inductance as seen from the roof bushings is approximately 4.4 microhenries per phase for 4.8 kv, 7.2 kv, 12.47 kv, and 13.8 kv equipments. For 2.4 kv and 4.16 kv equipments, this inductance is approximately 2.6 microhenries per phase.

3. Methods of frequency calculation which neglect the inductance of internal horizontal buses to which the capacitor units are directly connected are reasonably accurate and not overly pessimistic even when closely paralleled banks are considered.



4. When no resistance is purposely inserted, the damping can be neglected with satisfactory accuracy when calculating the magnitude of the first crest of the transient current for practical applications.

## REFERENCES

1. "Capacitor Installation at Newport News" by V. R. Parrack, E. L. Harder. AIEE Transactions, 1944, pp. 1165-72.

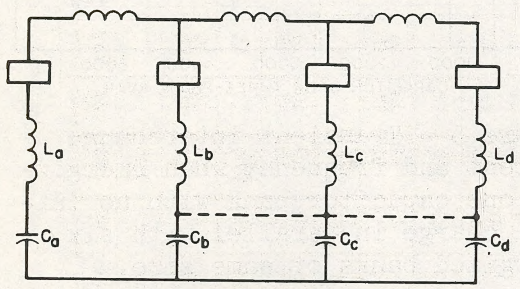



Figure 1-A - One-line diagram of typical parallel capacitor bank installation.

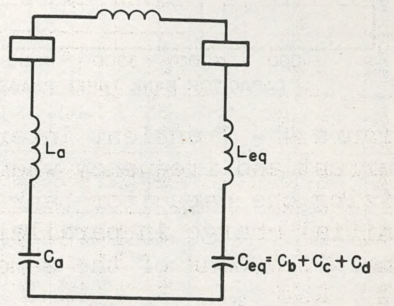



Figure 1-B - Equivalent circuit for switching bank (a) against banks (b), (c), and (d).

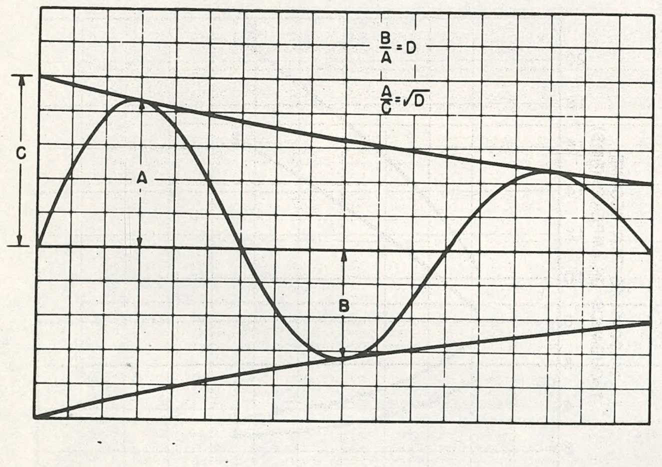



Figure 2 - Transient inrush current to a capacitor bank.

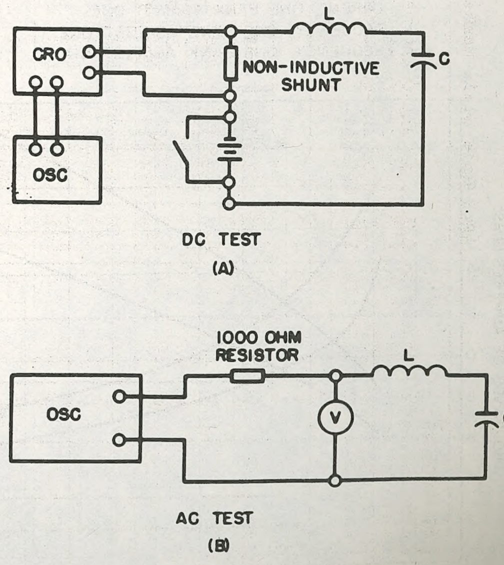
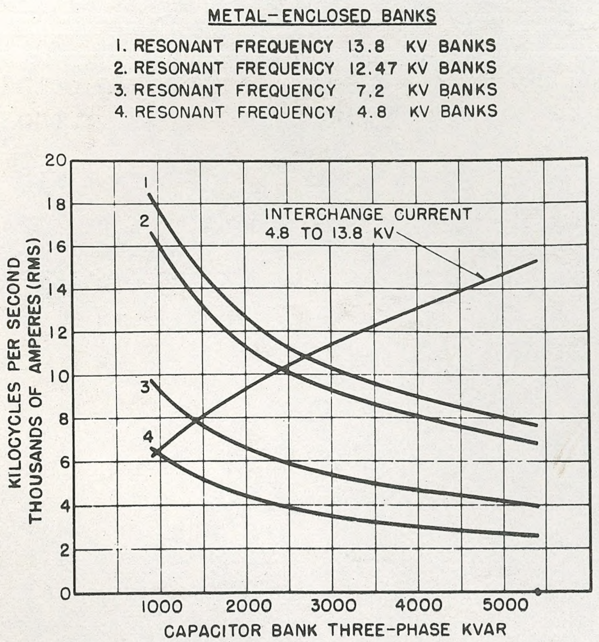



Figure 3 - Test methods.

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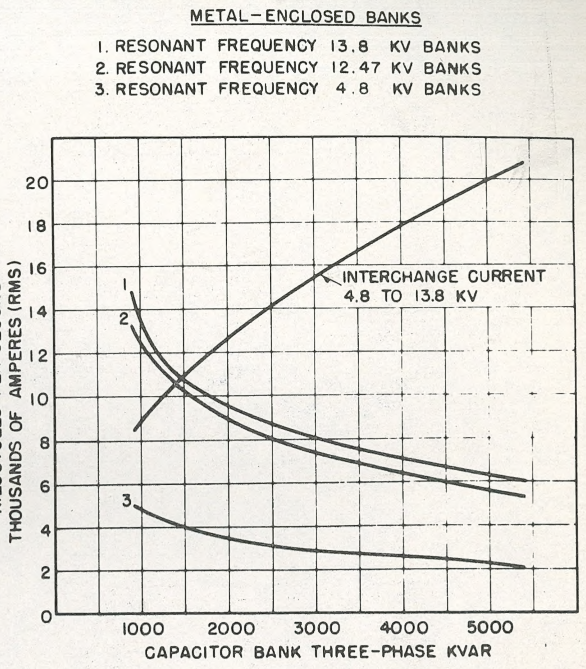


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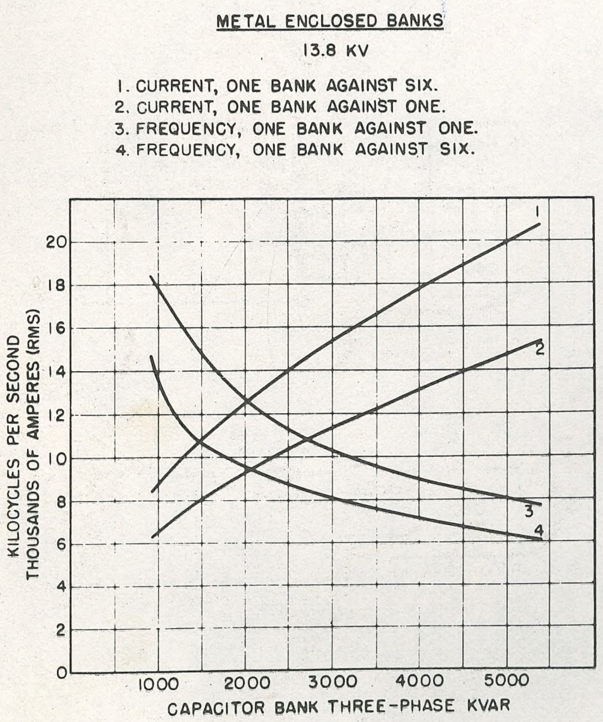


Figure 6 - Comparison of one bank against one and one bank against six.

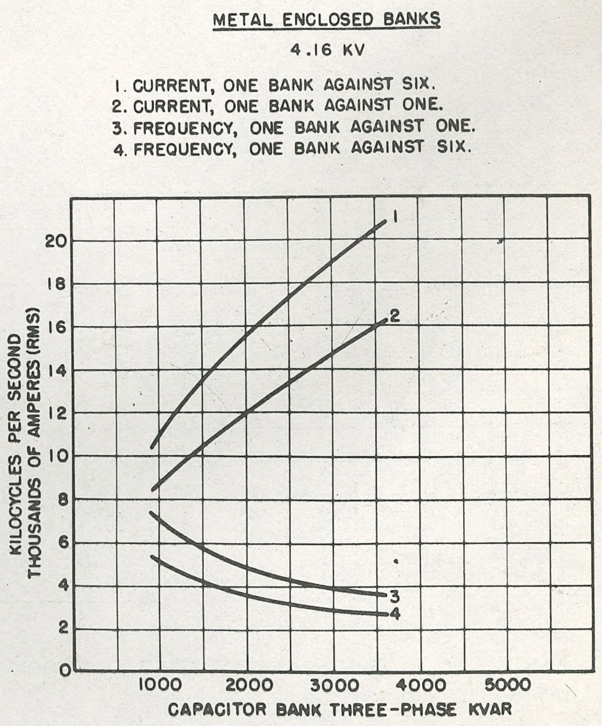


Figure 7 - Transient current and frequency of 4.16 kv metal-enclosed capacitor banks.


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